第 48 卷 第 2 期　　天 文 学 报　　Vol.48 No.2
2007 年 4 月　　ACTA ASTRONOMICA SINICA　　Apr., 2007

# 彗木相撞的可压缩分层流体模型*

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**摘要** 提出一个彗木相撞的修正了的线性模型. 在这个模型中, 将木星大气视为一个转动的、分层的、可压缩和无粘滞的流体层. 结果表明, 虽然大部分撞击能量仍然转换成内波能量, 但不能忽视可压缩效应.

**关键词** 彗星：个别, 行星与卫星：个别
**中图分类号**： P 185;　　**文献标识码**： A

## 1 引言

本文提出一种模型来处理彗木相撞所造成的波的传播. 作为初级近似, 我们采用线性模型. 为简化计, 我们忽略了非线性效应, 而在有的问题上它是非常重要的 (例如, 环的有限初始半径应该用撞击源区附近的非线性效应来说明). 哈勃空间望远镜 (HST) 观测到的在撞击地周围的 "环" 形构造最可能是大气波所致 [1]. 这些环具有常速度, 使人联想到它们是传播着的波. 本文在重力波的框架内讨论这个问题. 张可可等 [2,3] 和陈道汉等 [4] 曾经讨论过该问题并给出了一个不计流体可压缩性的模型. 在本文的模型中加入了压缩项并讨论由此造成的效应.

## 2 模型和基本方程

尝试着描述彗星撞击所产生的波动, 我们假设木星大气具有均匀的有限深度 $H$ 的连续分层和转动着的可压缩的流体层. 流体的粘滞性和有关的热效应则不予考虑. 还假定休梅克 – 列维彗星的碎片为密度均匀的球对称形状物体, 其对木星大气的冲撞所致的力学效应可用作在木星大气上的一个初始冲压来近似. 上述物理模型适于用文 [5] 的方法进行描述. 采用 $z$ 轴向上的柱坐标系 $(r,\theta,z)$, 以常角速度矢 $\Omega$ (沿 $z$ 轴) 表征的转动可压缩分层流体的流体动力学线性方程组为

$$\frac{\partial v_r}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial r}-fv_\theta=0\,, \tag{1}$$

2005-12-15 收到原稿, 2006-04-21 收到修改稿
* 国家自然科学基金 (批准号 10273024, 10573037) 和紫金山天文台小行星基金会资助项目

$$\frac{\partial v_\theta}{\partial t}+\frac{1}{\rho_0}\frac{\partial p^{'}}{r\partial\theta}+fv_r=0\,, \tag{2}$$

$$\frac{\partial w}{\partial t}+\frac{1}{\rho_0}\frac{\partial p^{'}}{\partial z}+\frac{\rho^{'}}{\rho_0}g=0\,, \tag{3}$$

$$\frac{\partial \rho^{'}}{\partial t}-\frac{\rho_0 N^2}{g}w-C_s^{-2}\frac{\partial p^{'}}{\partial t}=0\,, \tag{4}$$

$$\frac{\partial (rv_r)}{r\partial r}+\frac{\partial v_\theta}{r\partial\theta}+\frac{\partial w}{\partial z}+\frac{1}{\rho_0 C_s^2}\left(\frac{\partial p^{'}}{\partial t}-\rho_0 g w\right)=0\,, \tag{5}$$

其中 $v_r$ 、 $v_\theta$ 和 $w$ 表示扰动速度的分量， $p'$ 、 $\rho'$ 、 $\rho_0(z)$ 和 $g$ 分别为扰动压强、扰动密度、未扰密度和引力加速度， $C_s$ 为声速. 并且

$$f=2\Omega \tag{6}$$

是科里奥利 (Coriolis) 参量和

$$N^2=-g\left(\frac{d\rho_0}{\rho_0 dz}+\frac{g}{C_s^2}\right) \tag{7}$$

是 Vaisala 频率. 与 Gill[5] 描述的过程相似，我们可以导出下面两个关系， $w$ 和 $p'$ 的方程

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)\left(\frac{\partial w}{\partial z}-gC_s^{-2}w\right)=\rho_0^{-1}\frac{\partial}{\partial t}\left[\nabla_H^2-C_s^{-2}\left(\frac{\partial^2}{\partial t^2}+f^2\right)\right]p^{'}\,, \tag{8}$$

$$\frac{\partial^2 w}{\partial t^2}+N^2w=-\rho_0^{-1}\frac{\partial}{\partial t}\left(\frac{\partial p^{'}}{\partial z}+\frac{g}{C_s^2}p^{'}\right)\,, \tag{9}$$

其中

$$\nabla_H^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{r^2\partial\theta^2}\,. \tag{10}$$

## 3　正交模式

如果我们定义新变量

$$(V_r,V_\theta,W)=\rho_0^{\frac{1}{2}}(v_r,v_\theta,w),\qquad P=\rho_0^{-\frac{1}{2}}p^{'}\,, \tag{11}$$

并引入新变量 *Gamma*,

$$\Gamma=\frac{1}{2\rho_0}\frac{d\rho_0}{dz}+\frac{g}{C_s^2}=\frac{1}{2}\left(\frac{g}{C_s^2}-\frac{N^2}{g}\right)\,, \tag{12}$$

则方程 (8) 和 (9) 变成

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)\left(\frac{\partial W}{\partial z}-\Gamma W\right)=\frac{\partial}{\partial t}\left[\nabla_H^2-C_s^{-2}\left(\frac{\partial^2}{\partial t^2}+f^2\right)\right]P\,, \tag{13}$$

$$\frac{\partial^2 W}{\partial t^2}+N^2W=-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial z}+\Gamma P\right). \tag{14}$$

因为木星大气是一个流体薄层，所以可用浅水波模型 [6], 这意味着可以作流体静力学近似. 这时方程 (13) 不变并且仍为

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)\left(\frac{\partial W}{\partial z}-\Gamma W\right)=\frac{\partial}{\partial t}\left[\nabla_H^2-C_s^{-2}\left(\frac{\partial^2}{\partial t^2}+f^2\right)\right]P\,, \tag{15}$$

但是 (14) 式可近似为

$$N^2W=-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial z}+\Gamma P\right). \tag{16}$$

方程 (15) 和 (16) 具有形式为

$$W=\widehat{h}(z)\widetilde{w}(r,\theta,t),\quad P=\widehat{p}(z)\widetilde{\eta}(r,\theta,t) \tag{17}$$

的分离解，其中 $\hat{p}$ 和 $\hat{h}$ 满足

$$(C_e^{-2}-C_s^{-2})\widehat{p}=\frac{\mathrm{d}\widehat{h}}{\mathrm{d}z}-\Gamma\widehat{h}\,, \tag{18}$$

$$\frac{\mathrm{d}\widehat{p}}{\mathrm{d}z}+\Gamma\widehat{p}=-N^2\widehat{h} \tag{19}$$

而 $C_e$ 为速度量纲的分离常数. 与此同时， $\tilde{w}$ 和 $\tilde{\eta}$ 满足

$$\widetilde{w}=\frac{\partial\widetilde{\eta}}{\partial t}\,, \tag{20}$$

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)\widetilde{w}=C_e^2\frac{\partial}{\partial t}\nabla_H^2\widetilde{\eta}\,. \tag{21}$$

容易求出用分离变量表示的其他变量

$$\left.\begin{aligned} v_r&=\rho_0^{-\frac{1}{2}}\widehat{p}(z)\widetilde{v_r}(r,\theta,t)/g\\ v_\theta&=\rho_0^{-\frac{1}{2}}\widehat{p}(z)\widetilde{v_\theta}(r,\theta,t)/g\\ p^{'}&=\rho_0^{\frac{1}{2}}\widehat{p}(z)\widetilde{\eta}(r,\theta,t)\\ w&=\rho_0^{-\frac{1}{2}}\widehat{h}(z)\widetilde{w}(r,\theta,t)\\ h&=g(\rho^{'}-\frac{p'}{C_s^2})/\rho_0N^2=\rho_0^{-\frac{1}{2}}\widehat{h}(z)\widetilde{\eta}(r,\theta,t) \end{aligned}\right\}. \tag{22}$$

$\tilde{v}_r$, $\tilde{v}_\theta$ 和 $\tilde{\eta}$ 满足的方程，为

$$\partial\widetilde{v_r}/\partial t-f\widetilde{v_\theta}=-g\partial\widetilde{\eta}/\partial r,\partial\widetilde{v_\theta}/\partial t+f\widetilde{v_r}=-g\partial\widetilde{\eta}/r\partial\theta\,, \tag{23}$$

$$\partial\widetilde{\eta}/\partial t + H_e(\partial\widetilde{v_r}/\partial r + \widetilde{v_r}/r + r^{-1}\partial\widetilde{v_\theta}/\partial\theta) = 0\,. \tag{24}$$

其中 $H_e$ 是等价深度且满足 $H_e = C_e^2/g$. 按照文 [7], 我们可以设 $\ln\rho_0(z)$ 反比于深度 $z$, 亦即 $N =$ 常量. 若 $N$ 为常数且设 $\Gamma$ 与 $z$ 无关，则从 (18) 式和 (19) 式消去 $\hat{h}$ 后，我们有

$$\mathrm{d}^2\widehat{p}/\mathrm{d}z^2 + [N^2(c_e^{-2} - C_s^{-2}) - \Gamma^2]\widehat{p} = 0. \tag{25}$$

利用底部边界条件

$$\widehat{h} = 0 \quad \text{当}z = -H\text{时}, \tag{26}$$

我们有方程 (25) 的解

$$\widehat{p}(z) = \frac{C}{\cos mH}\left[\cos m(z+H) - \frac{\Gamma}{m}\sin m(z+H)\right], \tag{27}$$

$$\widehat{h}(z) = \frac{C(m^2+\Gamma^2)}{mN^2\cos mH}\sin m(z+H)\,, \tag{28}$$

$$m^2 = N^2(c_e^{-2} - C_s^{-2}) - \Gamma^2\,, \tag{29}$$

式中 $C$ 是任意常数. 我们选取 $C = \rho_0^{1/2}gH$, 理由是：正压模式 ($m \to 0$) 的解亦即 $\hat{p}_0(z) = \rho_0^{1/2}gH[1-\Gamma(z+H)]$, $\hat{h}_0(z) = \rho_0^{1/2}gH(\frac{1}{C_0^2} - \frac{1}{C_s^2})(z+H)$, $C_0^2 = gH$ 在 $C_s \to \infty$ 和 $\Gamma \to 0$ 时应当回复到不可压缩情形下的相应公式. 对于全部斜压模式而言，可以作刚盖近似 (rigid lid approximation), 表面条件为

$$\widehat{p} = \rho_0 g\widehat{h}, \quad \text{当}z = 0\text{时}\,, \tag{30}$$

应当代之以

$$\widehat{h} = 0, \quad \text{当}z = 0\text{时}. \tag{31}$$

于是，在这种情形下，我们有

$$C_n^2 = \frac{N^2C_s^2H^2}{N^2H^2 + C_s^2(n^2\pi^2 + \Gamma^2H^2)} \qquad n = 1, 2, ..., \tag{32}$$

式中 $C_n$ 是对应于第 $n$ 模式的相应 $C_e$ 值. 由方程 (15) 和 (16), 易得色散关系

$$\omega^2 = f^2 + \frac{N^2C_s^2(k^2+l^2)}{(m^2+\Gamma^2)C_s^2 + N^2} = f^2 + k_H^2C_n^2\,, \tag{33}$$

式中 $m = \dfrac{n\pi}{H}$ 结合文 [2,3] 对应于第 $n$ 模式的分离变量解 $\tilde{\eta}_n$, $\tilde{v}_{r,n}$ 和 $\tilde{v}_{\theta,n}$, 在轴对称流动的情形下，有

$$\left.\begin{aligned} w &= \rho_0^{-\frac{1}{2}} \sum_{n=0}^{\infty} \widehat{h}_n(z)\widetilde{w}_n(r,t) \\ p' &= \rho_0^{\frac{1}{2}} \sum_{n=0}^{\infty} \widehat{p}_n(z)\widetilde{\eta}_n(r,t) \\ h &= \rho_0^{-\frac{1}{2}} \sum_{n=0}^{\infty} \widehat{h}_n(z)\widetilde{\eta}_n(r,t) \\ v_r &= \rho_0^{-\frac{1}{2}} \sum_{n=0}^{\infty} \widetilde{v}_{r,n}(r,t)\widehat{p}_n(z)/g \\ v_\theta &= \rho_0^{-\frac{1}{2}} \sum_{n=0}^{\infty} \widetilde{v}_{\theta,n}(r,t)\widehat{p}_n(z)/g \end{aligned}\right\} . \tag{34}$$

## 4 初始扰动势能

在可压缩情形下，扰动势能加上表面波能量为

$$E_P = \int_0^{\infty} \left\{ \frac{1}{2}\rho_0(0) g\eta^2 + \int_{-H}^{0} \left[ \frac{1}{2}\rho_0 N^2 h^2 + \frac{1}{2}\rho_0^{-1} (\frac{p'}{Cs})^2 \right] \mathrm{d}z \right\} 2\pi r \mathrm{d}r \,, \tag{35}$$

其中 $\eta$ 是离开静止位置的表面位移 $h = g\left(\rho' - \dfrac{p'}{C_s^2}\right)\rho_0 N^2$. $E_P$ 由 3 部分构成：(1) 正压模式 ($n = 0$), 亦即表面波 (第一项); (2) 斜压模式 ($n = 1, 2, \cdots$) 亦即内波 (第二项); (3) 由可压缩性引入者 (第三项). 假设 $P(r, z, 0)$ 为初始压力扰动亦即初始冲力，则由 (34) 式我们可用正交模式表达给定的初始压力扰动

$$P(r,z,0) = \rho_0^{\frac{1}{2}} \sum_{n=0}^{\infty} \widehat{p}_n(z)\widetilde{\eta}_n(r,0) \,, \tag{36}$$

其中

$$\widetilde{\eta}_n(r,0) = \frac{\int_{-H}^{0} P(r,z,0)\widehat{p}_n(z)\mathrm{d}z}{\rho_0^{\frac{1}{2}} \int_{-H}^{0} \widehat{p}_n^2(z)\mathrm{d}z} \,. \tag{37}$$

对于正压模式，我们有

$$\eta \equiv h(r,0,0) = \rho_0^{-\frac{1}{2}}\widehat{h}_0(0)\widetilde{\eta}_0(r,0) = \frac{\left(1 - \dfrac{gH}{C_s^2}\right)}{\rho_0 g H \left(1 - \Gamma H + \dfrac{\Gamma^2 H^2}{3}\right)} \int_{-H}^{0} P(r,z,0)\left[1 - \Gamma(z+H)\right] \mathrm{d}z \,. \tag{38}$$

而对于可作刚盖近似的斜压模式，则有

$$\begin{aligned} \widehat{h}_n(z)\widetilde{\eta}_n(r,0) \quad = \quad & \frac{2}{\rho_0^{1/2} N^2} \frac{n\pi}{(-H)^2} \sin\left(\frac{n\pi z}{-H}\right) \int_0^{-H} P(r,z,0) \\ & \left[\cos\left(\frac{n\pi z}{-H}\right) + \frac{\Gamma H}{n\pi} \sin\left(\frac{n\pi z}{-H}\right)\right] \mathrm{d}z \,, \end{aligned} \tag{39}$$

$n=1,2,\cdots$, 于是

$$\begin{aligned} h(r,z,0) &= \rho_0^{-\frac{1}{2}}\sum_{n=1}^{\infty}\widehat{h}_n(z)\widetilde{\eta}_n(r,0) \\ &= \frac{2}{-H}\sum_{n=1}^{\infty}\left\{\frac{1}{\rho_0 N^2}\frac{n\pi}{-H}\int_0^{-H}P(r,z,0)\right. \\ &\quad \left.\left[\cos\left(\frac{n\pi z}{-H}\right)+\frac{\Gamma H}{n\pi}\sin\left(\frac{n\pi z}{-H}\right)\right]\mathrm{d}z\sin\frac{n\pi z}{-H}\right\}. \end{aligned} \tag{40}$$

将表达式 (38) 和 (40) 代入 (35) 并计算 (35) 式中的第三项，则可得当 $t=0$ 时的 $E_P$. 下面用两个情形来讨论能量问题.

### 4.1 情形 a

假设初始冲压的形式为

$$P(r,z,0)=P_0e^{-(z/R)^2-(r/R)^2}, \tag{41}$$

其中 $R$ 为实施撞击的彗星碎片的有效半径，并将表达式 (41) 代入 (38) 和 (40), 我们可得

$$\begin{aligned} \eta\equiv\eta(r,0,0) &= \frac{\left(1-\dfrac{gH}{C_s^2}\right)P_0\left[(1-\Gamma H)\sqrt{\pi}R+\Gamma R^2\right]e^{-(r/R)^2}}{2\rho_0 gH\left(1-\Gamma H+\dfrac{\Gamma^2H^2}{3}\right)}\Phi\left(\frac{H}{R}\right) \\ &\sim \frac{\left(1-\dfrac{gH}{C_s^2}\right)P_0\left[(1-\Gamma H)\sqrt{\pi}R+\Gamma R^2\right]e^{-(r/R)^2}}{2\rho_0 gH\left(1-\Gamma H+\dfrac{\Gamma^2H^2}{3}\right)}, \end{aligned} \tag{42}$$

(其中，误差函数的值 $\Phi(\dfrac{H}{R})\sim1$, 因为 $\frac{H}{R}$ 相当大) 和

$$\begin{aligned} h(r,z,0) &= \frac{\pi R(\sqrt{\pi}+\Gamma R)P_0e^{-(r/R)^2}}{-H^2\rho_0N^2}\sum_{n=1}^{\infty}ne^{-n^2\pi^2R^2/4H^2}\sin\frac{n\pi z}{-H} \\ &\sim -\frac{2(\pi^{1/2}+\Gamma R)P_0e^{-(r/R)^2}}{R\rho_0N^2}\frac{\sinh\left[\dfrac{2H}{R}\left(1+\dfrac{z}{H}\right)\right]}{\sinh(2H/R)}. \end{aligned} \tag{43}$$

注意到

$$\frac{1}{2}\int_0^{\infty}\int_{-H}^{0}\rho_0^{-1}\left[\frac{P(r,z,0)}{C_s}\right]^2\mathrm{d}z\cdot2\pi r\mathrm{d}r=\frac{P_0^2R^3\pi^{3/2}}{8\sqrt{2}\rho_0C_s^2},$$

则将 (42) 式和 (43) 式代入 (35) 式后给出

$$\begin{aligned} E_P &= \frac{\pi^2R^2P_0^2}{4\rho_0}\left[\frac{R^2}{4gH^2}\left(\frac{1-\Gamma H+\Gamma R\pi^{-1/2}}{1-\Gamma H+\dfrac{\Gamma^2H^2}{2}}\right)^2\left(1-\frac{gH}{C_s^2}\right)^2+\right. \\ &\quad \left.\frac{\left(1+\Gamma R\pi^{-1/2}\right)^2}{RN^2}+\frac{R}{\sqrt{8\pi}C_s^2}\right]. \end{aligned} \tag{44}$$

因为木星大气的主要成分是 $H_2$, 我们可以近似地将它看作是由双原子气体构成的大气薄层. 于是，我们有 $C_s^2=\frac{7}{5}gH_s$, $N^2=\frac{2}{7}gH_s^{-1}$, $\Gamma=\frac{3}{14}H_s^{-1}$, 式中的 $H_s$ 表示木星大气的标高. 现在，我们假定撞击的彗星碎片的全部动能转换成初始扰动势能和表面波能量，亦即 $E_P$, 于是我们有

$$\frac{1}{2}\rho_c\left(\frac{4}{3}\pi R_c^3\right)U^2=\frac{\pi^2R^2P_0^2}{4\rho_0}$$
$$\left[\frac{R^2}{4gH^2}\left(\frac{1-\Gamma H+\Gamma R\pi^{-1/2}}{1-\Gamma H+\frac{\Gamma^2H^2}{2}}\right)^2\left(1-\frac{gH}{C_s^2}\right)^2+\frac{(1+\Gamma R\pi^{-1/2})^2}{RN^2}+\frac{R}{\sqrt{8\pi}C_s^2}\right], \quad (45)$$

其中 $\rho_c$ 和 $R_c$ 分别表示彗星碎片的密度和半径，$U$ 为撞击在木星表面时的相对速率. 可以选取 $H_s$ 为 25.8 km 并且 $C_s$ 、$N$ 和 $\Gamma$ 对应地为 969 km/s 、$1.7\times10^{-2}/s$ 和 $8.3\times10^{-6}m^{-1}$. 又，我们取 $R\sim R_c=1\,\mathrm{km}$, $\rho_c=0.5\,\mathrm{g/cm^3}$, $U=60\,\mathrm{km/s}$, $H=8\,\mathrm{km}$, $g=2.6\times10^3\,\mathrm{cm/s^2}$, $\rho_0=10^{-3}\,\mathrm{g/cm^3}$, 于是由 (45) 式可得 $P_0=2.09\times10^7\,\mathrm{bar}$, 而且还能求得表面波能量对总波动能量的初始比率

$$q_1=\frac{\frac{R^2}{4gH^2}\left(\frac{1-\Gamma H+\Gamma R\pi^{-1/2}}{1-\Gamma H+\frac{\Gamma^2H^2}{2}}\right)^2\left(1-\frac{gH}{C_s^2}\right)^2}{\frac{R^2}{4gH^2}\left(\frac{1-\Gamma H+\Gamma R\pi^{-1/2}}{1-\Gamma H+\frac{\Gamma^2H^2}{2}}\right)^2\left(1-\frac{gH}{C_s^2}\right)^2+\frac{\left(1+\Gamma R\pi^{-1/2}\right)^2}{RN^2}+\frac{R}{\sqrt{8\pi}C_s^2}}=$$
$$2.62\times10^{-5}, \quad (46)$$

以及流体可压缩性所致的能量对总波动能量的比

$$q_2=\frac{\frac{R}{\sqrt{8\pi}C_s^2}}{\frac{R^2}{4gH^2}\left(\frac{1-\Gamma H+\Gamma R\pi^{-1/2}}{1-\Gamma H+\frac{\Gamma^2H^2}{2}}\right)^2\left(1-\frac{gH}{C_s^2}\right)^2+\frac{\left(1+\Gamma R\pi^{-1/2}\right)^2}{RN^2}+\frac{R}{\sqrt{8\pi}C_s^2}}=$$
$$6.08\times10^{-5}. \quad (47)$$

容易看出，虽然可压缩性的确影响能量的分布，内波的能量仍然是主部.

### 4.2 情形 b

我们假定初始冲压取另一种形式

$$P(r,z,0)=P_0e^{z/R-r/R},$$

对应地，有

$$\eta \equiv \eta(r,0,0) = \frac{\left(1-\frac{gH}{C_s^2}\right)P_0 e^{-r/R}}{\rho_0 g H} R \frac{(1-\Gamma H+\Gamma R)(1-e^{-H/R})-\Gamma H(e^{-H/R})}{(1-\Gamma H+\frac{\Gamma^2 H^2}{3})}, \quad (48)$$

$$h(r,z,0) = -\frac{P_0 e^{-r/R}}{\rho_0 N^2}\frac{2}{-H}(\frac{1}{R}+\Gamma)\sum_{n=1}^{\infty}\frac{n\pi(-H)}{n^2\pi^2+(-H/R)^2}[1-(-1)^n e^{-H/R}]\sin\frac{n\pi z}{-H}$$
$$= -\frac{P_0 e^{-r/R}}{\rho_0 N^2}(\frac{1}{R}+\Gamma)e^{z/R}, \quad (49)$$

$$E_P = \frac{\pi R^4 P_0^2}{4\rho_0 g H^2}\frac{\left(1-\frac{gH}{C_s^2}\right)^2[(1-\Gamma H+\Gamma R)(1-e^{-H/R})-\Gamma H(e^{-H/R})]^2}{(1-\Gamma H+\frac{\Gamma^2 H^2}{3})^2} +$$
$$\frac{\pi R P_0^2(1+\Gamma R)^2(1-e^{-2H/R})}{8\rho_0 N^2}+\frac{\pi R^3 P_0^2}{8\rho_0 C_s^2}(1-e^{-2H/R})$$
$$\sim \frac{\pi R P_0^2}{4\rho_0}\left[\frac{R^3}{gH^2}\frac{\left(1-\frac{gH}{C_s^2}\right)^2(1-\Gamma H+\Gamma R)^2}{\left(1-\Gamma H+\frac{\Gamma^2 H^2}{3}\right)^2}+\frac{(1+\Gamma R)^2}{2N^2}+\frac{R^2}{2C_S^2}\right], \quad (50)$$

$$\frac{1}{2}\rho_c\left(\frac{4}{3}\pi R_c^3\right)U^2 = \frac{\pi R P_0^2}{4\rho_0}\left[\frac{R^3}{gH^2}\frac{\left(1-\frac{gH}{C_s^2}\right)^2(1-\Gamma H+\Gamma R)^2}{\left(1-\Gamma H+\frac{\Gamma^2 H^2}{3}\right)^2}+\frac{(1+\Gamma R)^2}{2N^2}+\frac{R^2}{2C_S^2}\right]. \quad (51)$$

这导致 $P_0 = 5.22\times10^7\,\mathrm{bar}$ 和

$$q_1 = \frac{R^3\left(1-\frac{gH}{C_s^2}\right)^2(1-\Gamma H+\Gamma R)^2}{R^3\left(1-\frac{gH}{C_s^2}\right)^2(1-\Gamma H+\Gamma R)^2+gH^2\left[\frac{(1+\Gamma R)^2}{2N^2}+\frac{R^2}{2C_S^2}\right]\left(1-\Gamma H+\frac{\Gamma^2 H^2}{3}\right)^2} =$$
$$2.10\times10^{-4}, \quad (52)$$

以及

$$q_2 = \frac{\frac{R^2}{2C_S^2}}{\frac{R^3}{gH^2}\frac{\left(1-\frac{gH}{C_s^2}\right)^2(1-\Gamma H+\Gamma R)^2}{\left(1-\Gamma H+\frac{\Gamma^2 H^2}{3}\right)^2}+\frac{(1+\Gamma R)^2}{2N^2}+\frac{R^2}{2C_S^2}} = 3.03\times10^{-4}. \quad (53)$$

## 5 结论

为了描述彗木相撞时在木星表面产生的波动，我们假设木星大气为均匀高度 $H$ 的转动分层流体，并且计入了流体可压缩性的影响．当受到彗星撞击时，木星大气的垂直扰动速度分量 $w$ 和扰动压力 $p'$ 满足方程 (8) 和 (9).

浅水波模型可以应用于木星大气层，这意味着可以作流体静力学近似．使得方程 (9) 中的二阶时间导数项可以去掉，亦即对应于频率 $\omega$ 小于 $N$．在这种情况下，方程组可以分离变量求解.

讨论流体可压缩性的影响，最好是从能量的角度考虑．通过具体例子计算撞击能量的分配，发现可压缩性的贡献大致与表面波的相当，而内波能量仍是能量分配的主要部分.

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# A Compressive Stratified Fluid Model of the Comet's Collision with Jupiter

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**Abstract** In order to explain the waves arising from the collisions of Comet Shoemaker-Levy 9 with Jupiter, we present a revised linear model in which the gravity wave propagation associated with the impacts is caused by an initial impulse and the Jovian atmosphere is considered as a rotating stratified, compressive and inviscid fluid layer.

By means of a procedure similar to that adopted in Gill's book (1982), two derived equations (8) and (9) involving only the perpendicular component of perturbation velocity and the perturbation pressure can be obtained. In terms of the new variables defined by (11) and (12), the equations turn into (13) and (14).

As pointed out by Dowling (1995), the Jovian atmosphere is a thin sheet of fluid and the shallow-water model can be used. It also means that the hydrostatic approximation

can be used and the basic equations become (15) and (16). These equations have separable solutions of the form (17).

It is stressed that we take an initial impulsive pressure prescribed at the surface of Jupiter as the initial condition rather than an initial surface deformation. This approach is expedient for the mathematical formulation of the problem, for instance, the vertical displacement of a fluid particle can be expressed by an integral transform of the potential energy and thereupon can be calculated.

It is also shown by the calculations that although the contribution of the compressibility in the participation of impact energy is larger than that of the surface waves, which indicates the effect of compressibility can not be ignored, the energy of the internal waves is still the main part, i.e. most of the impacting energy is converted into the energy of internal waves.

**Key words comet: individual, planet and satellite: individual**